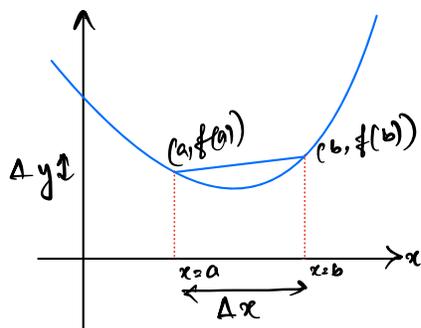


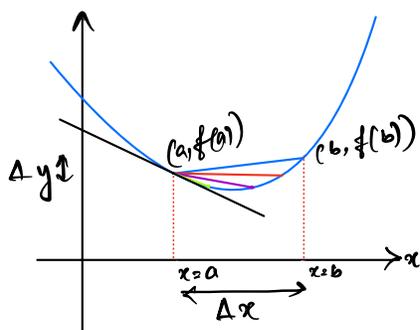
Defining the Derivative:

To find the average rate of change between two distinct points, we find the slope of the secant line.



$$\begin{aligned} \text{slope} &= \frac{\Delta y}{\Delta x} \\ \text{(secant)} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{f(b) - f(a)}{b - a} \end{aligned}$$

If we want to find the rate of change at one single point, then we move the other point along the curve towards the required point.



$$\begin{aligned} \text{slope} &= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \\ \text{(tangent)} &= \lim_{b \rightarrow a} \frac{f(b) - f(a)}{b - a} \end{aligned}$$

Defⁿ: Let $f(x)$ be defined in an interval containing a

Then the derivative (or the slope of the tangent) at $x=a$ is given by

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

or

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

} Instantaneous rate of change relative to a .

Ex. $f(x) = x^2$, Find $f'(x)$ at $x = 3^a$ & then find the equation of the tangent to $f(x)$ at $x = 3$.

→ $f(x) = x^2$

$$\text{Then } f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

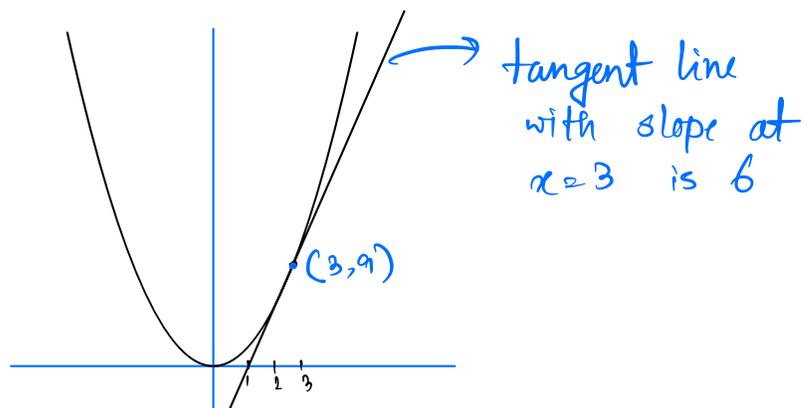
Replace a by 3 , so

$$f'(3) = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 3^2}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x+3)(x-3)}{(x-3)} \quad [\text{Note: } x-3 \neq 0]$$

$$= \lim_{x \rightarrow 3} (x+3) = 3+3 = 6$$



In this example, tangent line is $y = 6x + b$, where the tangent passes through $(3, 9)$ point.

$$\text{So, } 9 = 6(3) + b \Rightarrow b = -9$$

$$\text{So, tangent line is, } y = 6x - 9$$

Let $f(x) = e^x$. Find $f'(x)$, if $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$.

$$\rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{e^x [e^h - 1]}{h}$$

$$= e^x \cdot \lim_{h \rightarrow 0} \frac{e^h - 1}{h}$$

$$= e^x \cdot 1 = e^x$$

$$\textcircled{*} \text{ given, } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$